Maneuvering Loads and the V-n Diagram

The V-n, velocity-versus-load diagram, Figure 1, describes the relationship between an aircraft’s speed, its longitudinal (pitch axis) maneuvering capability, and its structural strength. The positive-g, maximum lift line indicates how aggressively, at any airspeed, we can apply aft pressure to pitch an aircraft to change its flight path without stalling the wings or doing damage through excessive loads. The maximum lift line shows how our excess margin of nose-up pitch control (in other words, the load factor, or g, available in reserve) diminishes as we slow down, disappearing finally at the 1-g stall. At that point, in a normal aircraft, we can only pitch down.

The limits represented by the parabolic maximum lift line are also physiological, and dramatically so. If you pull too hard, climb the lift line too high, and stay there too long, the blood starts leaving your brain. Your face becomes strikingly woeful in the video; your vision collapses from gray to black. Then consciousness shuts down. When the blood returns and the lights come on, your short-term memory is an empty hole. Amnesia is common enough in centrifuge trials that the USAF believes that many fighter pilots who experience g-induced loss of consciousness (G-LOC) never realize it.

Each V-n diagram is for a specific aircraft weight and wing configuration (lift devices in or out): see Figure 2. Indicated airspeed is generally used. Calibrated airspeed is sometimes used since it corrects IAS for position errors caused by the placement of the pitot and static sources, and by gauge errors within the airspeed indicator.
itself. An aircraft will stall at the same calibrated airspeed regardless of altitude. If the gauge error depends only on airspeed, the aircraft will stall at the same indicated airspeed, regardless of altitude.

The maximum lift line, or $C_{L_{\text{max}}}$ boundary, takes its parabolic shape from the fact that lift is a function of velocity squared (because lift is proportional to dynamic pressure, $q$, which is itself proportional to $V^2$). You can draw the lift line based purely on an aircraft’s 1-g stall speed at a given weight. At least you can for speeds to about Mach 0.3. Above that, compressibility effects take over, $C_{L_{\text{max}}}$ declines, and the slope of the curve decreases.

Load factor, $n$, ($n = \text{Lift}/\text{Weight}$) is what’s read on the g meter. In normal, 1-g equilibrium flight, lift equals weight. In 2-g turning or looping flight, the aircraft produces lift equal to double its weight. Some of that extra lift goes to generate a centripetal force that accelerates the aircraft toward the center of an arc. Flight at more than 1 g is always associated with a pitch rate.

As you increase your pitch rate at a given airspeed, your g-load increases until you reach the maximum lift line and stalling angle of attack. Actually, before you hit the lift line you usually hit a buffet boundary, as airflow separated from the wing and fuselage starts belaboring the tail. In aircraft with personality issues, the buffet boundary might be severe, or the aircraft might have a pitch-up tendency or a wing rock before reaching maximum coefficient of lift, $C_{L_{\text{max}}}$, in which case the operational boundary is defined by those characteristics rather than by an actual stall break.

The lift line represents the maximum load factor obtainable at the corresponding velocity. In a conventional aircraft you can’t fly to the left of the line because the wing will stall first. The aircraft unloads itself. You might exceed the lift line briefly by a quick charge, since dynamic effects can allow airfoils to sustain lift momentarily at greater than normal stalling angle of attack, if angle of attack is increased at a high rate per second. But then you’d just fall back into the envelope as the momentary, extra lift disappears.

**Stall speed goes up by the square root of the load factor.** So at 2 g, for example, stall speed goes up by a factor of 1.4 (since $\sqrt{2} = 1.4$).

![Figure 2](image-url)
Maneuvering Speed, $V_A$

As defined by the V-n diagram, maneuvering speed, $V_A$, is the maximum speed at which an aircraft in symmetrical flight at the specified flight weight and configuration will stall (unload) before exceeding limit load and sustaining possible structural damage. Aircraft are therefore aerodynamically g-limited by the lift line up to maneuvering speed, and structurally g-limited by the load line above it. Maneuvering speed is also the maximum speed for turbulent air penetration, although a speed somewhat less—fast enough to avoid stall yet slow enough to diminish the loads experienced—is usually recommended. (In an aircraft subjected to a sharp vertical gust of given intensity, the increase in structural load—and thus the acceleration the pilot feels—varies directly with airspeed.)

At speeds above roughly Mach 0.3, $C_{L_{\text{max}}}$ begins to decrease. Mach number depends on altitude, so indicated $V_A$ increases with altitude because you have to go faster to generate equivalent lift at the lower $C_{L_{\text{max}}}$.

Symmetrical flight means the aircraft isn’t rolling and isn’t yawed. The load is symmetrical across the span. That’s not the case in a rolling pull-up, however, where the rising wing experiences a higher load than the wing going down (the rising wing is lifting more because of the camber change; the descending wing lifting less). The g meter in the fuselage reads only the average load, as in Figure 3.

Rolling pull-ups became a problem with the F4U Corsair gull-wing fighter in World War II. Reportedly, pilots would roll with aileron as they pulled out after a ground attack run, hoping to place the aircraft’s protective armor plating between them and the answering ground fire. They sometimes went past limit load in the roll and came home with bent wings along with the usual shell holes.

$V_A$ and limit load (as measured at the fuselage) therefore decrease if the aircraft is rolling. The rolling motion could come from aileron deflection, or from aggressive rudder input causing a roll couple (as in a snap roll). Aircraft flight manuals that specify a maximum limit load for rolling pull-ups typically place it at two-thirds to three-quarters of the symmetrical limit load. If you settle on a conservative two-thirds, a 6-g aerobatic aircraft has a rolling pull-up (and snap roll) limit of 4 g. To keep things in round numbers, the “rolling” $V_A$, as calculated for the aircraft weight, would be about twenty percent less than the symmetrical $V_A$.

By the same logic a large aircraft certified under FAR 25.337(b) with the minimum allowed limit load of 2.5 would be restricted to a 1.65-g rolling pull-up, assuming that Mach buffet, caused by the transonic acceleration of the airflow over the wing as angle of attack is increased, doesn’t occur first.

Unload before rolling if you’re in a high-g situation and need to level the wings.

The above not withstanding, maneuvering speed is usually defined—without regard to asymmetrical loads—as the maximum speed at which full or abrupt combined control movements can be made without damaging the aircraft. The FAA’s AC 61-23C, “Pilot’s Handbook of Aeronautical Knowledge,” says that “any combination of flight control usage, including full deflection of the controls, or gust loads created by turbulence should not create an excessive air load if the airplane is operated below maneuvering speed.” According to the Navy, “Any combination of maneuver and gust cannot create damage due to excess airload when the airplane is below the maneuver speed.”

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Maneuvering Loads, High-G Maneuvers

The NTSB has pointed out that this broader definition, although widespread among pilots, is incorrect. Engineers consider each axis separately in designing for the air loads accompanying an abrupt, full control input at maneuvering speed. “Full inputs in more than one axis at the same time and multiple inputs in one axis are not considered in designing for these \( V_A \) flight conditions.”

The particular “multiple inputs” that prompted NTSB comment were the rudder reversals leading to a yaw over swing followed by a final reversal that destroyed the vertical tail of American Airlines Flight 587 on November 12, 2001. (Under FAR 25.351, rudders are tested for sudden displacement in a single direction at a time, and then returned to neutral, at speeds up to design dive speed.)

So here’s a conservative, inclusive, legalistic mouthful: Maneuvering speed, \( V_A \), is the maximum speed, at a given weight and configuration, at which any one (and only one) flight control surface can be abruptly and fully deflected—not to include rapid control surface reversals—without causing aircraft damage.

Simple Formulas

These formulas help define the relationships between aircraft weight, speed, and load.

(1) 1-g Stall Speed vs. Aircraft Weight: Knowing the 1-g stall speed, \( V_S \), at any weight gives you the 1-g stall speed for any other weight:

\[
\sqrt{\frac{\text{New Weight}}{\text{Known Weight}}} \times (\text{Known } V_S) = \text{New } V_S
\]

(2) Stall Speed and Load Factor: Stall speed goes up as the square root of the load factor, \( n \). To find the accelerated stall speed, \( V_{S_{\text{acc}}} \), for a given load factor:

\[
V_{S_{\text{acc}}} = V_S \sqrt{\text{Load factor, } n}
\]

(3) Maneuvering Speed, \( V_A \). Given the 1-g stall speed, to determine an aircraft’s maneuvering speed at maximum takeoff weight for upright flight in its category, use the formula above and substitute \( V_A \) for \( V_{S_{\text{acc}}} \). Insert a load factor of:

- 3.8 for Normal & Commuter but see FAR Part 23.337(1).
- 4.4 for Utility
- 6 for Aerobatic
- FAR Part 25.337(b), 2.5 minimum

(4) Maximum Aerodynamic Load Factor for a Given Airspeed: The highest load factor you can pull at a given airspeed is based on the 1-g stall speed, \( V_S \), at the aircraft’s actual weight. You can use this to plot the lift line in the V-n diagram:

\[
\left( \frac{\text{Airspeed}}{V_S} \right)^2 = \text{Load factor, } n
\]

(5) Maneuvering Speed vs. Aircraft Weight: Like other \( V \) speeds calculated on the basis of aircraft weight, maneuvering speed, \( V_A \), goes down as aircraft weight goes down. If the aircraft is under max gross takeoff weight, the allowable limit and ultimate limit loads don’t change (so interpret the g meter as usual). Only the corresponding \( V \) speeds change as the maximum lift line shifts toward the left. Although the total lift force that the wing has to develop at limit load is less at lower weights, and the stress on the wing is less, individual aircraft components still weigh the same. Things like engine mounts, battery trays, luggage racks, chandeliers (it happens), and landing gear up-lock systems may not be designed to withstand more than their component weight times limit load. At lower gross weights that load can be reached at lower speeds because the wing doesn’t have to produce as much lift. Since it doesn’t have to work as hard, it won’t stall until after the limit load is exceeded.

To calculate \( V_A \) at reduced aircraft weight:

\[
\sqrt{\frac{\text{New Weight}}{\text{Max Takeoff Weight}}} \times (\text{Max Takeoff } V_A) = \text{New } V_A
\]

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Corner Speed

$V_A$ is also known as corner speed, $V_C$, especially by fighter pilots, for whom it has tremendous tactical significance. $V_C$ is the speed for maximum instantaneous turn performance without exceeding structural limits. “Instantaneous” is used because an aircraft might not have the thrust necessary to sustain $V_C$ under the elevated induced drag of maneuvering at high angle of attack. Sustained corner speed has a lower value.

**Turn rate goes to maximum at corner speed.**

That’s because turn rate is proportional to $n/V_T$ ($n$ is load factor; $V_T$ is true airspeed). Combinations of high $g$ and low airspeed favor turn rate. Corner speed is the lowest airspeed at which maximum structural $g$ is possible. Flying at maximum structural $g$ at any speed in excess of $V_C$ causes turn rate to decrease.

A high turn rate is obviously important to fighter pilots because it allows them to achieve firing solutions in a turning fight.

**Turn radius goes to minimum at corner speed.**

Turn radius is proportional to $V_T^2/n$. Therefore radius is minimized by high $g$ and low airspeed. Again, corner speed is the lowest speed for the highest allowable structural $g$.

The top of Figure 4 suggests how the radius decreases as load factor rises. It decreases quickly at first, but then the rate of change per $g$ slows down. Note that flying at maximum structural $g$ at any speed in excess of $V_C$ causes the turn radius to increase.

Low wing loading (aircraft weight/wing area) favors maneuverability. At a given $C_L$, minimum turn radius and maximum rate come when the aircraft is light. Higher air density (thus lower altitude) also favors maneuverability.

**Pull-ups**

Corner speed is spoken of in discussions of turning flight, but remember that turning isn’t limited to the horizontal plane. The pull-up you may find yourself in during a nose-low unusual-attitude recovery is a vertical turn. Here, achieving minimum radius may be crucial. Figure 4 shows the relationship between corner speed and turn radius in a pull-up.

**The strategy for a low-altitude, maximum-performance, minimum-radius, limit-load pull-up is to pull to and maintain $C_{L_{max}}$ (indicated by initial buffet, angle of attack indicator, stick shaker, fly-by-wire $g$ limiter, unacceptable wing rock) until reaching corner speed, then remain at limit load until recovery.**

The problem is blasting through $V_C$, (or starting the recovery past $V_C$) so you’ll want to have the drag devices out, pending approval from the POH/AFM. In propeller aircraft, that includes power back and flat pitch for more drag. Lowering the landing gear might blow off the doors or lead to a partial extension, but that could be a good trade. In a jet, what you do with power could depend on the pitching moment associated with power change. With fuselage-mounted engines, throttles would normally come back. But retarding power on an aircraft with
wing-pylon-mounted engines creates a nose-down pitching moment. Some speed brakes do the same. The POH/AFM is the guide. At low altitude, Mach buffets and Mach-related trim effects presumably would not be a factor.

Critics of the use of corner speed as part of recovery procedure point out that the speed varies with aircraft weight, and there’s the “potential that pilots could fixate on obtaining and maintaining corner speed, while delaying or overlooking implementation of other recovery techniques, and result in [sic] unnecessary loss of altitude during a nose low recovery. Exposing pilots to the concept of corner speed and radius of turn as a basis for understanding why it may be necessary to increase speed in order to recover from a nose low, low altitude upset is beneficial. However, incorporating a corner speed into recovery procedure, we feel is inappropriate.”

Sounds like a sensible objection.

Student behavior suggests that the most common error is to be too gentle on the aircraft in the initial part of a dive recovery. For fear of overstressing the aircraft, pilots are reluctant to add normal acceleration (g’s) to longitudinal acceleration (the aircraft’s increasing speed), so they bring in the g slowly. But the induced drag created by the increased lift necessary for normal acceleration also acts as a brake on longitudinal acceleration. Pull smoothly—no yanking into an accelerated stall that actually lowers pitch rate—but if ground avoidance is at stake don’t hesitate in getting to the maximum g (i.e., maximum pitch performance and minimum radius) the flight condition allows.

Rolling is a limiting flight condition. If necessary, level the wings before a pull up. The asymmetrical load caused by aileron deflection, added to a pull-up load, can overstress the wing. Again, an aircraft’s \( V_A \) and g-meter limit load decrease when rolling. And as pilots generally don’t recognize, high adverse yaw generated by large aileron deflection while pulling could lead to high sideslip angles and bending stresses on the vertical tail.

A wings-level pull-up is also more efficient, since the entire load is applied to lifting the nose to the horizon, and not partly to turning.

Pull-ups and Phugoids

The hands-off phugoids we fly at the beginning of our flight program demonstrate that a longitudinally stable aircraft will try to pull out of a dive by itself. The altitude consumed will depend on the true airspeeds and load factors attained.

At a given g at any instant, the radius of either a pilot-induced or a pure phugoid-induced pull-up varies with the square of the airspeed. As a result, for example, if you enter twice as fast, but your load factors remain identical, you’ll consume four times the altitude.

(The g that actually matters in maneuvering performance is “radial g,” explained farther on. Radial g depends both on the load factor seen on the g meter and on aircraft attitude.)

The phugoid-generated load factor depends on the design and balance characteristics of the control system, but more essentially on the difference between the airspeed attained and the trim speed. Remember that during the phugoid the aircraft maintains a constant angle of attack. At a constant angle of attack, lift goes up as the square of the increase in airspeed. For example, if we trim for 100 knots in normal flight (1 g) and reach 200 knots in a phugoid dive recovery, airspeed will be double the trim speed and the load factor will hit a theoretical 4 g. If we accelerate to 140 knots, it’s a 1.4 increase in airspeed over trim. \(1.4^2 = 2\); thus a load factor of 2 g.

A pilot can overstress an aircraft in a dive by a pull on the stick in addition to the aircraft’s natural phugoid. Again, the load generated by the phugoid depends on trim speed versus airspeed. The required pull, or g-limiting push, depends on how this load compares to limit load.

The classic disaster pattern consists of the horizontal stabilizers failing downward first if the pilot pulls too hard. When they fail, the aircraft suddenly pitches nose down, and the wings fail downward because of the sudden negative load.
Lift Vector, Radial G, and the Split-s

In a level turn, as shown at the top of Figure 5, the tilted lift vector has two vectoral components, a vertical one equal to and opposite aircraft weight, and a horizontal one pointing toward the center of the turn. While the pilot feels (and the g meter reads) loads in the direction of the tilted lift vector, the horizontal, centripetal force that’s actually turning the aircraft—its radial g—has a lower value.

As bank angle increases in coordinated, constant-altitude flight, radial g grows. Past 90 degrees of bank, the lift vector starts pointing toward the earth, and radial g gets a boost from gravity. The result can be up to a 1-g gain in radial g in inverted flight at the top of a loop.

For a given load factor (g on the meter), pointing the lift vector above the horizon decreases radial g and pitch rate; pointing it below the horizon increases radial g and pitch rate. The increased radial g available in inverted attitudes can help win dogfights, but it’s a trap for untrained pilots. It’s why, at a given airspeed and applied g, positive (nose toward your head) pitch rates when flying inverted are higher than positive pitch rates when flying upright, and why pulling back on the stick as a reaction to the confusion of inverted flight so quickly brings the nose down and the airspeed up. The resulting split-s entry (half loop from inverted), especially if provoked by an inexperienced pilot who releases his aft control pressure out of contrition once the nose starts down, then changes heart and pulls some more, can quickly take the aircraft outside the envelope of the V-n diagram. That’s when it rains aluminum.

In a nose-down, inverted unusual-attitude recovery, the most important thing is to get the lift vector pointed back above the horizon. Except at extreme nose-down attitudes, that means rolling upright rather than pulling through in a split-s. In brief: When inverted, push to keep the nose from falling further. Roll the lift vector skyward with full aileron while removing the push force as you pass through knife-edge. Then raise the nose.

Just so you know, maximum structural instantaneous turn performance happens while pulling maximum g, at corner speed, inverted.